

Examiner:

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- (a) State the functional equation for $\zeta(s)$.
- (b) Describe the locations of the zeros and poles of $\zeta(s)$, giving brief justifications.
- (c) Let $N(T)$ count the number of zeros of $\zeta(s)$ in the region

$$\{s = \sigma + it : 0 \leq \sigma \leq 1 \text{ and } 0 \leq t \leq T\}.$$

Sketch a proof that

$$N(T) = \frac{T}{2\pi} \log \frac{T}{2\pi} - \frac{T}{2\pi} + O(\log T).$$

Your sketch should be no longer than half a page, and should focus on the key steps. You do not need to go into any details for technical calculations or estimates.

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- (a) Show that

$$\pi(x) \gg \frac{x}{\log x}.$$

- (b) State the prime number theorem. What do you know about the error term, both assuming the Riemann Hypothesis and unconditionally?
- (c) Explain (in no more than half a page) why the distribution of zeros of the zeta function affects the distribution of prime numbers.